Progressive 3D Reconstruction of Planar-Faced Manifold Objects with DRF-Based Line Drawing Decomposition

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Abstract—This paper presents an approach for reconstructing polyhedral objects from single-view line drawings. Our approach separates a complex line drawing representing a manifold object into a series of simpler line drawings, based on the degree of reconstruction freedom (DRF). We then progressively reconstruct a complete 3D model from these simpler line drawings. Our experiments show that our decomposition algorithm is able to handle complex drawings which are challenging for the state of the art. The advantages of the presented progressive 3D reconstruction method over the existing reconstruction methods in terms of both robustness and efficiency are also demonstrated.

Index Terms—3D reconstruction, degree of reconstruction freedom, decomposition, line drawing, optimization-based

1 INTRODUCTION

A line drawing is the simplest and most direct way of illustrating a 3D object. The human vision system is able to interpret 2D line drawings as 3D objects with no difficulty. Emulating this ability is an important but challenging research topic for machine vision. 3D reconstruction from line drawings benefits various applications such as the design of flexible sketching interfaces for conceptual designers [1]–[6], and sketch-based 3D object retrieval [7]–[9].

Similar to previous 3D reconstruction methods in [3], [10]–[14], we tackle the problem of 3D reconstruction from single line drawings with an optimization-based approach. Optimization-based 3D reconstruction methods determine the depth values (z-coordinates) of the vertices of a line drawing from the solution that optimizes a certain objective function. The main problem with the existing methods is that they become less robust and less efficient when a line drawing is complex.

This work focuses on improving the performance of optimization-based 3D reconstruction from complex line drawings. We intend to avoid the search in a high-dimensional space in the optimization process. We present an efficient degree-of-reconstruction-freedom (DRF) based algorithm (Section 4), which decomposes a complex line drawing into a series of simpler parts with small DRFs (Figures 1(a) and (b)). As we will show, compared to the existing line drawing decomposition technique [13], our algorithm is much more efficient and is able to successfully handle more line drawings with high complexity.
In addition, we propose an estimate-and-optimize strategy for progressive reconstruction of the separated simpler parts (Section 5). Our method starts with 3D reconstruction of an initial part (Fig. 1(c)), and sequentially reconstructs other parts (Fig. 1(d)) based on the already reconstructed parts. The latter provides a good initial setting for the reconstruction of subsequent ones, thus greatly improving the robustness of optimization-based 3D reconstruction from line drawings.

2 RELATED WORK

Computational interpretation of line drawings has spanned more than four decades. A full review of this topic is beyond the scope of this paper. See [15]–[17] for insightful surveys. The earliest work towards 3D reconstruction from single line drawings is line labeling, which focuses on finding a set of consistent labels from a line drawing and testing its correctness and realizability [18]–[20]. Line labeling is useful to detect some perceptual relations in a line drawing, but it does not explicitly give the 3D shape represented by the line drawing. The methods based on linear programming [21]–[23] reconstruct a 3D model by solving a linear system which is built from a set of geometrical conditions that the model must fit. In general, linear programming has difficulty tolerating sketching errors that often exist in a line drawing.

Modern methods of 3D reconstruction from line drawings are often optimization based. The seminal work by Marill [14] presented a very simple regularity for 3D reconstruction, i.e., minimizing the standard deviation of the angles (MSDA) in the reconstructed objects so that a 2D line drawing can be inflated into a 3D shape. Motivated by MSDA, Brown and Wang [24] proposed to minimize the standard deviation of the segment magnitudes (MSDSM), and Shoji et al. [25] presented the criterion of minimizing the entropy of angle distribution (MEAD). MSDA, MSDSM, and MEAD can only recover simple objects from line drawings. Leclerc and Fischler’s method [11] considers not only MSDA but also the planarity constraint on the faces of the object (planarity constraint is a powerful property and many methods have been proposed to find faces from a line drawing [26]–[31]). This method performs better than MSDA, MSDSM, and MEAD. Later, Lipson and Shpitalni [3] extended Leclerc and Fischler’s method by using more constraints like line parallelism, line verticality, isometry, etc., enabling the reconstruction of more complex objects than Leclerc and Fischler’s. Based on the works [3], [11], [14], Turner et al. recovered simple planar 3D objects from scenes [32]. Shesh and Chen applied Lipson and Shpitalni’s algorithm to their sketching system in [33]. Liu et al. proposed a plane-based optimization method in [12] which finds desired objects in a much lower dimensional search space by enforcing a set of linear constraints on the unconstrained optimization problem. Their method can tackle 3D reconstruction of more complex objects, but fail to obtain an expected 3D object when the degree of reconstruction freedom of an object is large [13].

Our work might be the most relevant to that by Liu et al. [13], which also presented a line drawing decomposition algorithm. As illustrated in Fig. 2, their algorithm firstly decomposes a line drawing into a set of simpler line drawings (LD1 and LD2 in Fig. 2(b)) along internal faces (f∗, marked in red in Fig. 2(a)), then reconstructs 3D shapes (O1 and O2 in Fig. 2(c)) from the resulting simpler line drawings independently, and finally merges the 3D shapes together and fine-tunes the merged 3D model. By decomposing a complex line drawing into simpler line drawings, their method avoids the search in a high dimension space.

One of the limitations of the method in [13] is that the decomposition algorithm based on finding internal faces is NP-complete. To make this algorithm run in tolerable time, a predefined maximum search depth has to be set. Therefore, it cannot find an internal face when the number of its edges is larger than the threshold. Another deficiency of [13] is that the 3D shapes are reconstructed independently from the decomposed line drawings, without using the geometrical relationships between the decomposed line drawings. This might cause artifacts in the subsequent merging step, e.g., non-planar faces in the complete model in Fig. 2(d) after merging O1 and O2 via f∗ 1 and f∗ 2. This is mainly because while f∗ 1 and f∗ 2 are consistent in the decomposed line drawings, the corresponding faces in O1 and O2 might become inconsistent due to independent reconstruction. This justified their extra fine-tuning step, which, however, does not always work well and is time consuming.

Our progressive reconstruction method based on an estimate-and-optimize strategy shares some resemblances to those proposed in [3], [10], and [34], which employ certain geometrical properties to provide an initial guess for the subsequent optimization-based 3D reconstruction. For example, to obtain a preliminary approximation of the object, Lipson and Shpitalni [3] assume that the edges in the line drawing have three main axis directions. The approach of Company et al. [10] is
applicable only to the input line drawing representing a (quasi-)normalon, while the method proposed by Lee and Fang [34] requires the availability of at least one cubic corner in a desired object. Our approach does not require such special geometrical properties of an underlying object in the line drawing. This is achieved by decomposing the line drawing into multiple parts and progressively optimizing each part, instead of solving the optimization problem on the entire line drawing. Our approach is thus more general and is applicable to a wider class of objects.

3 ASSUMPTIONS, PREPROCESSING, AND TERMINOLOGY

Similar to [13], our paper focuses on 3D reconstruction of a large class of common planar-faced solids, called manifolds. On the surface of a manifold, every point has a neighborhood topologically equivalent to an open disk in the 2D Euclidean space [35]. A line drawing in this paper is assumed to be an orthographic projection of the edges of a 3D planar-faced manifold in a generic view, with hidden lines and vertices visible.

Preprocessing. The face topology is very crucial for the line drawing decomposition and reconstruction. Given a line drawing, we use the algorithm in [28] to find its face topology. Here, the face topology denotes the set of circuits that represent all the faces of the 3D object. For example, the line drawing in Fig. 3(a) has 15 faces, as shown in Fig. 3(b). See another example in Figures 4(c) and (d). Artificial lines (see examples in Fig. 3(c)), added by the designer, are used to indicate the coplanarity of two circuits in solid modeling [36], [28]. Detecting artificial lines is an easy task according to the connection between an artificial line and the edges it connects to [13]. After removing the artificial lines, a line drawing becomes two or more separated line drawings (Fig. 3(d)).

For better understanding of the contents in the following sections, we here give the definitions of the terms that appear in the rest of the paper.

**Definition 1.** Let a line drawing be the projection of a 3D object. The minimum number of depths (z-coordinates) that can uniquely define this 3D object is called the degree of reconstruction freedom (DFR) for the line drawing.

This definition comes from [12], where DFR is used to find a search space for 3D reconstruction. Instead, in our paper, it is used to decompose complex line drawings. Now, let us analyze the DFR for a simple line drawing shown in Fig. 4(a). Assume that the line drawing is the precise projection of a 3D planar-faced object. Thus, all 3D vertices on the same face are coplanar. For example, all the vertices $v_{1-4}$ are on the plane defined by $a_1x + b_1y + c_1 - z = 0$, which passes through the face $f_1 = (v_1, v_2, v_3, v_4)$. Next, we can show that the 3D object is defined if $z_1, z_2, z_3$, and $z_5$ are given, where $z_i$ is the depth value of vertex $v_i$. When $z_1, z_2$ and $z_3$ are known, the 3D plane $a_1x + b_1y + c_1 - z = 0$ is defined. Then, $z_4$ can be calculated by $z_4 = a_1x_4 + b_1y_4 + c_1$. Since $z_4$ is known and $z_1$ and $z_5$ are given, $v_8$ is determined because it lies on the plane defined by $v_1, v_5$ and $v_4$. Analogously, the depths of all other vertices can be determined. On the other hand, it is obvious that the given depth values of only three vertices are not sufficient to determine a unique 3D object for this line drawing. Therefore the DFR for this drawing is 4.

**Definition 2.** Let a line drawing be $LD = (V, E, F)$ where $V$, $E$, and $F$ are the sets of vertices, edges, and faces of $LD$, respectively. A partial line drawing, or a part of $LD$, is formed by one or more connected faces
denoted by $\mathcal{LD}_p = (V_p, E_p)$ with $V_p \subseteq \mathcal{V}$ and $E_p \subseteq \mathcal{E}$. A neighboring face of $\mathcal{LD}_p$ is a face that has at least one edge (but not all) in $\mathcal{LD}_p$. Given two parts $\mathcal{LD}_{p_1}$ and $\mathcal{LD}_{p_2}$, let $F(\mathcal{LD}_{p_1})$ and $F(\mathcal{LD}_{p_2})$ be the sets of the faces in $\mathcal{LD}_{p_1}$ and $\mathcal{LD}_{p_2}$, respectively; let $E(\mathcal{LD}_{p_1})$ and $E(\mathcal{LD}_{p_2})$ be the sets of the edges in $\mathcal{LD}_{p_1}$ and $\mathcal{LD}_{p_2}$, respectively. $\mathcal{LD}_{p_1}$ and $\mathcal{LD}_{p_2}$ are called two neighboring parts if $F(\mathcal{LD}_{p_1}) \cap F(\mathcal{LD}_{p_2}) = \emptyset$ and $E(\mathcal{LD}_{p_1}) \cap E(\mathcal{LD}_{p_2}) \neq \emptyset$.

Fig. 4(b) shows two parts of the line drawing in Fig. 4(a). The face $f_1$ in $\mathcal{LD}_{p_2}$ is a neighboring face of the part $\mathcal{LD}_{p_1} = \{f_4 \cup f_5\}$. In Fig. 4(e), $\mathcal{LD}_{p_1}$ and $\mathcal{LD}_{p_2}$ are two neighboring parts, which share the same circuit (in blue) but share no face.

**Definition 3.** A 4DRF part is a part whose DFR is four. A manifold is called a 4DRF manifold if the DFR for its line drawing is four.

Fig. 5 shows four examples of 4DRF line drawings. Their complexities vary, though they have the same DFR. The line drawing in Fig. 5(c) has only two faces. It is the simplest 4DRF line drawing. The line drawing shown in Fig. 5(a) has 6 faces and is more complex. It is easy to show that a 4DRF line drawing or 4DRF partial line drawing has at least two faces. The cuboid represented by the line drawing in Fig. 5(a) is a 4DRF manifold.

**Definition 4.** A 4DRF-extended part of a 4DRF part $\mathcal{LD}_p$, denoted as $4DRF Ext(\mathcal{LD}_p)$, is the part which contains $\mathcal{LD}_p$ and one neighboring face of $\mathcal{LD}_p$, which has two non-collinear edges in $\mathcal{LD}_p$. The largest-4DRF-extended part of a 4DRF part $\mathcal{LD}_p$, denoted as $Largest4DRF Ext(\mathcal{LD}_p)$, is the 4DRF part which has the largest number of faces among the parts, each of which contains $\mathcal{LD}_p$.

Note that the DFRs of $\mathcal{LD}_p$, $4DRF Ext(\mathcal{LD}_p)$, and $Largest4DRF Ext(\mathcal{LD}_p)$ are all 4. As shown in Fig. 4(b), given a part $\mathcal{LD}_{p_1}$, $\mathcal{LD}_{p_1} = \{f_4 \cup f_5\}$, a 4DRF-extended part of $\mathcal{LD}_{p_1}$ is the partial line drawing which contains all the faces in $\mathcal{LD}_{p_1}$ and the face $f_1$, since $f_1$ passes through two non-collinear edges in $\mathcal{LD}_{p_1}$. For a 4DRF part of a line drawing, it may have no 4DRF-extended part. As shown in Fig. 4(e), each neighboring face of $\{f_3 \cup f_7\}$ shares less than two non-collinear edges with $\{f_3 \cup f_7\}$ and hence the part $\{f_3 \cup f_7\}$ has neither 4DRF-extended part nor largest-4DRF-extended part. It is worth noting that a largest-4DRF-extended part does not necessarily represent a manifold, though this work focuses the decomposition of line drawings representing manifolds only.

Given a 4DRF part $\mathcal{LD}_p$, the following procedure $4DRF Ext_F(\mathcal{LD}_p)$ (Procedure 1) obtains one possible 4DRF-extended part:

**Procedure 1 $4DRF Ext_F(\mathcal{LD}_p)$**
1. $\mathcal{LD}_{tmp} \leftarrow \mathcal{LD}_p$;
2. add a neighboring face of $\mathcal{LD}_p$ to $\mathcal{LD}_{tmp}$ if this face has two non-collinear edges in $\mathcal{LD}_{tmp}$;
3. $4DRF Ext(\mathcal{LD}_p) \leftarrow \mathcal{LD}_{tmp}$;

**Procedure 2 Largest4DRF Ext_F(\mathcal{LD}_p)**
1. $\mathcal{LD}_{tmp} \leftarrow \mathcal{LD}_p$;
2. $\mathcal{LD}_{tmp} \leftarrow 4DRF Ext_F(\mathcal{LD}_{tmp})$;
3. if $\mathcal{LD}_{tmp} \supset \mathcal{LD}_p$, goto Step 2;
4. $\mathcal{LD}_{tmp} \leftarrow \mathcal{LD}_{tmp}$, goto Step 2;
5. $Largest4DRF Ext_F(\mathcal{LD}_p) \leftarrow \mathcal{LD}_{tmp}$.

Obviously, the output of $4DRF Ext_F(\mathcal{LD}_p)$ is a valid 4DRF-extended part of $\mathcal{LD}_p$ only if $4DRF Ext F(\mathcal{LD}_p) \supset \mathcal{LD}_p$ (denoted $\mathcal{LD}_p$) is satisfied. Based on $4DRF Ext_F(\mathcal{LD}_p)$, starting from a 4DRF part, called a seed, $Largest4DRF Ext F(\mathcal{LD}_p)$ can be obtained by the procedure $Largest4DRF Ext F(\mathcal{LD}_p)$ (Procedure 2).

**Definition 5.** A neighboring part of a part $\mathcal{LD}_p$ is called a conditional-1DRF neighboring part of $\mathcal{LD}_p$ if its DFR is 1 when the 3D coordinates of all the vertices of $\mathcal{LD}_p$ are known. A conditional-1DRF neighboring part of $\mathcal{LD}_p$ is called the largest-conditional-1DRF neighboring part of $\mathcal{LD}_p$ if it has the minimum number of faces among all the conditional-1DRF neighboring parts of $\mathcal{LD}_p$. Given a neighboring face $f$ of $\mathcal{LD}_p$, the largest-conditional-1DRF neighbor part of $\mathcal{LD}_p$ containing $f$ is denoted as $LargestCon1DRF(\mathcal{LD}_p, f)$.

We take the line drawing shown in Fig. 4(e) to illustrate the two terms in Definition 5. Assume that a 3D shape has been reconstructed from the part $\mathcal{LD}_{p_1} = \{f_1 \cup f_2 \cup f_3 \cup f_4 \cup f_5\}$. From Definition 2, there exist multiple neighboring parts of $\mathcal{LD}_{p_1}$, such as $\{f_6\}$, $\{f_6 \cup f_7\}$, and $\mathcal{LD}_{p_2} = \{f_6 \cup f_7 \cup f_8 \cup f_9 \cup f_{10}\}$. All of them are also conditional-1DRF neighboring parts of $\mathcal{LD}_{p_1}$, since when the 3D shape of $\mathcal{LD}_{p_1}$ is known, the depth of one vertex (not in $\mathcal{LD}_{p_1}$) of such a part can uniquely define the 3D position of this part.

Given a part $\mathcal{LD}_p$ and one of its neighbor faces $f$, $LargestCon1DRF(\mathcal{LD}_p, f)$ can be obtained by the following procedure $LargestCon1DRF F(\mathcal{LD}_p, f)$:
LargestCon

For this example, the algorithm compared to that by the algorithm in [13].

Algorithm 1 DRF-based line drawing decomposition.

1. Input: A line drawing $G_D = (V_0, E_0, F_0)$ where $V_0$, $E_0$, and $F_0$ are the sets of vertices, edges, and faces, respectively.
2. Build the dual graph $G = (V, E)$ of $G_D$.
3. Label($v$) $\leftarrow 0$ for every vertex $v \in V$; $i \leftarrow 0$; $V_u \leftarrow \emptyset$; $E_u \leftarrow \emptyset$; let $G_u = (V_0, E_0)$.
4. $G_D \leftarrow 0$; find a set of seeds $G_{sp}$ from $G - G_u$.
5. if $G,D = 0$; goto Step 9.
6. for each seed $G_{sp}$, in $G_{sp}$ do
   7. (a) $G_{tmp} \leftarrow 4DRF_{Ext,F}(G_{sp})$ (Procedure 1); (b) if $G_{tmp} \supset G_{sp}$ then
      8. flag1a $\leftarrow 1$ and goto Step 10;
   end if
   9. if flag1a $= 0$ then
      10. $G'_{pi} \leftarrow \text{Largest4DRF}_{Ext,F}(G_{tmp})$ (Procedure 2);
      11. $i \leftarrow i + 1$; Label($v$) $\leftarrow 1$ for every vertex $v \in G'_{pi}$;
      12. add $v$ to $V_u$ for every vertex $v \in G'_{pi}$; add $e$ to $E_u$ for every edge $e \in G'_{pi}$;
      13. if $G_u = G$; goto Step 22;
      14. AdjVer($G_u$) $\leftarrow$ all the neighboring vertices of $G_u$;
      15. flag1c $\leftarrow 0$;
      16. check if there is another neighboring face of $G_u$ in $G_D$ containing two non-collinear edges in $(G_u \cup \bar{e})$.
      17. if yes, flag1c $\leftarrow 1$ and goto Step 19;
      18. if flag1c $= 0$, goto Step 3;
      19. $G'_{pi} \leftarrow \text{LargestCon4DRF}_{Ext,F}(G_{u}, \bar{e})$ (Procedure 3);
      20. $i \leftarrow i + 1$; Label($v$) $\leftarrow 1$ for every vertex $v \in G'_{pi}$;
      21. add $v$ to $V_u$ for every vertex $v \in G'_{pi}$; add $e$ to $E_u$ for every edge $e \in G'_{pi}$; goto Step 13;
      22. if $i > 1$ Output: $(G'_{p1}, ..., G'_{pi})$ else Output: $G_{p1}$.
but slightly different decompositions. We will give a thorough evaluation of these two algorithms in Section 6.

The detailed decomposition steps are summarized in Algorithm 1 and illustrated in Figure 7. In Step 1, the dual graph $G$ (Fig. 7(b)) of the input line drawing $G_0$ is built to facilitate the design of the decomposition algorithm. $G_u$ in Step 2 is a subgraph of $G$, and is used to keep the currently decomposed parts. We use a binary label $\text{Label}(v)$ for each vertex $v$ in $G$ to denote whether $v$ is in the obtained parts in $G_u$.

In Steps 2–22, to have a concise description, a sign and its over-hatted sign are used to denote the two corresponding entities in $G$ and $G_0$, respectively. For example, $v_j$ is a vertex in $G$ while $\hat{v}_j$ is the corresponding face in $G_0$; $G_{sp}$ denotes two adjacent vertices in $G$ while $G_{sp}$ denotes the corresponding two adjacent faces in $G_0$; $G-G_u$ denotes the remaining subgraph by removing $\mathcal{V}_u$ and the edges connecting the vertices in $\mathcal{V}_u$ from $G$.

Through Steps 3–12 our algorithm finds the parts belonging to largest-4DRF-extended parts. A binary label $\text{flag}_l$ is used to check if a largest-4DRF-extended part can be found. More specifically, in Step 3 a seed set $G_{sp}$ is constructed for the generation of a largest-4DRF-extended part. In this step, all the pairs of adjacent faces, each pair sharing a trihedral vertex (i.e., a vertex passed through by three faces) in $G-G_u$, are selected as the seeds and added into $G_{sp}$. See an example of such a seed (adjacent vertices 1 and 5) in Fig. 7(c). We use such pairs as the seeds since a trihedral vertex usually exists in simple manifolds that are the targeted decomposition results, while a vertex passed through by more than three faces (say, $v_5$ in Fig. 4(c)) often indicates a place where two simple manifolds are merged into a complex manifold. The part growing step ($\text{Largest4DRFE}xt_F$) starting from a seed usually gives a desired largest-4DRF-extended part, e.g., $\{1,2,3,4,5\}$ in Fig. 7(d). Note that if the seed set constructed in Step 3 is empty, Algorithm 1 will output $G-G_u$ as the last part and stop.

Steps 14–17 check whether $G_u$ has a largest-conditional-1DRF neighboring part using a binary label $\text{flag}_{lc}$. According to $\text{flag}_{lc}$ in Steps 16 and 18, the algorithm goes on to find a largest-conditional-1DRF neighboring part of $G_u$ (Steps 19–21) or another largest-4DRF-extended part (Steps 3–12). Fig. 7(e) shows a largest-conditional-1DRF neighboring part $\{6,7,8,9\}$ of the part $\{1,2,3,4,5\}$. Fig. 7(f) gives two decomposed parts resulting from Algorithm 1.

### 4.2 Complexity

We now analyze the complexity of the decomposition algorithm. Suppose a line drawing $G_0$ has $N_v$ vertices, $N_e$ edges, and $N_f$ faces. Assume that every face in $G_0$ has less than $K_e$ edges or adjacent faces, and $G_0$ is decomposed into $K_p$ parts. Then the average number of faces in a part is $N_f/K_p$.

The main computation is carried out by Steps 3–21. In Step 3, the algorithm tests each pair of two adjacent faces in a subgraph of $G_0$ (i.e., $G-G_u$ in $G$) and adds those sharing a trihedral vertex into the set of seeds. This step takes less than $O(K_eN_f)$ time and obtains less than $K_eN_f$ seeds. The number of seeds determines the number of loops in Steps 6 and 7. Step 6 calls the procedure $\text{4DRFE}xt_F()$ to test whether the current seed can grow or not. In procedure $\text{4DRFE}xt_F()$, to test if a given neighboring face $f$ of $\mathcal{LD}_p$ has two non-collinear edges in $\mathcal{LD}_p$, we first find all the edges in $f$ that are also in $\mathcal{LD}_p$ and then check if there are two non-collinear edges among them. The former and the latter are bounded by $O(K_eN_v)$ and $O(K_e^2)$, respectively. Therefore the time of one execution of $\text{4DRFE}xt_F()$ is bounded by $O(N_f(K_eN_v+K_e^2)) = O(K_eN_vN_f)$. Considering Step 7 takes much less time than Step 6, the computation of Steps 5–8 is bounded by $O(K_e^2N_vN_f^2)$. The main computation of Steps 4–9 is determined by Steps 5–8, and thus the computation of Steps 4–9 is also bounded by $O(K_e^2N_vN_f^2)$. The main computation of Steps 10–13 is determined...
by the procedure \textit{Largest4DRFExt}_F() in Step 10, which is approximately equal to repeated execution of 4DRFExt\_F() for \( N_f/K_p \) times. Thus, the time of one execution \textit{Largest4DRFExt}_F() is bounded by \( O(K_r N_c N_f^2/K_p) \). Then, the total time of obtaining a largest-4DRF-extended part (Steps 3–13) is bounded by \( O(K_r N_c N_f^2) + O(K_r N_c N_f^2/K_p) = O(K_r^2 N_c N_f^2) \).

Next, we discuss the complexity of Steps 14–21. In these steps, the algorithm first checks if \( G_u \) has a conditional-1DRF neighboring part or not (Steps 15–17), and then finds a conditional-1DRF neighboring part (Steps 19–21) or another largest-4DRF-extended part (Steps 3–13). The complexity of Steps 14–21 is thus approximately to the time taken by Steps 3–13. Finally, the algorithm conducts Steps 3-13 or Steps 14–21 for \( K_p \) times. Considering \( K_r \) is constant, we have the complexity of Algorithm 1 bounded by \( O(K_r^2 N_c N_f^2 K_p) = O(K_r^2 N_c N_f^2) \), which is polynomial.

In fact, from our experiments, we find that the time spent by Algorithm 1 and by the line drawing pre-processing step (typically less than 1 second), can be almost neglected compared to the time taken by the 3D reconstruction algorithm (see Fig. 12(a); typically from 10 to 30 seconds).

5 Progressive 3D Reconstruction

DRF-based decomposition, introduced in the previous section, results in a set of simpler line drawings. A straightforward solution to recover a 3D model from such line drawing parts is to first reconstruct 3D shapes from individual parts of the line drawing independently and then integrate the reconstructed 3D shapes into a complete 3D model. However, as shown in Fig. 2(d), such an approach might easily cause reconstruction artifacts. To address this problem we introduce a novel progressive 3D reconstruction algorithm. Below we first give the algorithm details in Section 5.1 and then the implementation details in Section 5.2 and 5.3.

5.1 Algorithm

Our algorithm takes an estimate-and-optimize strategy and sequentially reconstructs 3D shapes from line drawing parts one by one based on already reconstructed parts. The algorithm is summarized in Algorithm 2.

Due to the progressive nature our algorithm would accumulate reconstruction errors. To reduce such errors we use the following scheme to derive a reconstruction sequence (Step 2), i.e., an ordered list of the decomposed parts, denoted as \( (G_{p_1}, \ldots, G_{p_N}) \), where \( N \) is the number of the decomposed parts of the input line drawing. As illustrated in Fig. 8 we first build a graph \( G' \) with each vertex representing a line drawing part and each edge connecting two neighboring parts. To determine the initial part \( G_{p_1} \), we estimate the reconstruction error starting from each vertex \( v_{G'_{p_i}} \) (i.e., \( G_{p_i}' \) of Algorithm 1) and take the one with the minimum error as \( G_{p_1} \). Mathematically it is formulated as follows:

\[
G_{p_i} = \arg \min_i \text{Err}(v_{G'_{p_i}}) = \sum_{j=1,j\neq i}^{N} \text{dis}(v_{G'_{p_i}}, v_{G'_{p_j}}),
\]

where \( \text{dis}(v_{G'_{p_i}}, v_{G'_{p_j}}) \) denotes the length of the shortest path between two vertices in the graph \( G' \), which makes \( G_{p_i} \) roughly correspond to the center of the graph \( G' \). In case there are two or more vertices with the same minimum Err, we pick \( G_{p_i} \) as the one corresponding to the part with the maximum \( N_{ppe}/N_e \), where \( N_{ppe} \) denotes the number of pairs of parallel edges in a part, and \( N_e \) denotes the number of the edges in the part, since a part with more parallel lines usually corresponds to a more regular 3D shape and its reconstruction is thus more robust. We determine the order of the rest of the parts using classic breadth-first search in \( G' \), starting from the vertex corresponding to \( G_{p_1} \).

Algorithm 2 3D reconstruction from a complex line drawing.

1. decompose an input line drawing with Algorithm 1;
2. determine the reconstruction sequence \( (G_{p_1}, \ldots, G_{p_N}) \);
3. reconstruct a 3D \( O_{p_1} \) shape from \( G_{p_1} \);
4. for \( i = 2 \) to \( N \) do
5. a) estimate a rough 3D shape \( \widetilde{O}_{p_i} \) with the information of \( (G_{p_1}, \ldots, G_{p_{i-1}}) \cup \{O_{p_1}, \ldots, O_{p_{i-1}}\} \);
6. b) reconstruct a 3D shape \( O_{p_i} \) for \( G_{p_i} \) based on \( \widetilde{O}_{p_i} \cup\{O_{p_1}, \ldots, O_{p_{i-1}}\} \) and \( (G_{p_1}, \ldots, G_{p_{i-1}}) \);
7. end for

In Step 3, a 3D shape is reconstructed from the initial part \( G_{p_1} \). Steps 4–7 sequentially reconstruct 3D shapes from the subsequent parts, where \( \widetilde{O}_{p_i} \) is the rough 3D shape estimated from the 3D information.
of the reconstructed 3D shapes \( \{O_{p_1}, O_{p_2}, \ldots, O_{p_{i-1}}\} \)
and \( \{G_{p_1}, \ldots, G_{p_i}\} \). With \( O_{p_i}, \{O_{p_1}, \ldots, O_{p_{i-1}}\} \),
and \( \{G_{p_1}, \ldots, G_{p_i}\} \), a 3D shape \( O_{p_i} \) is recovered by an
optimization-based algorithm. As we will show in Section 6 that, by estimating rough 3D shapes first, the
reconstruction algorithm usually converges much faster.

Step 6 can be achieved by adapting existing optimization-based 3D reconstruction algorithms like
those proposed in [3], [10], and [12]. Specifically, \( O_{p_i} \) can be reconstructed by minimizing the following objective function:

\[
\Phi(Z_{p_i}) = \sum_{j=1}^{N_c} w_j \phi_j(Z_{p_1}, \ldots, Z_{p_{i-1}}, Z_{p_i}), \quad (2)
\]

where \( Z_{p_i} \) is the set of the \( z \)-coordinates of the vertices in the \( i \)th part which are unknown and
their initial values are obtained from \( O_{p_i}, Z_{p_1}, \ldots, Z_{p_{i-1}} \),
denote the known \( z \)-coordinates of the vertices in the
\( i - 1 \) already reconstructed parts, \( \phi_j \), \( 1 \leq j \leq N_c \),
are the \( N_c \) constraints, i.e., image regularities, derived
from all the \( i \) parts, and \( w_j \) is a weighting factor. We
adopt the most commonly used image regularities [3],
[10]–[12], including face planarity, line parallelism, line
collinearity, skewed facial symmetry, isometry, corner
orthogonality, and minimizing the standard deviation of
angles in the reconstructed objects.

Below we give the implementation details of how to
reconstruct an initial 3D shape (Step 3) in Section 5.2,
and how to estimate an initial 3D shape for a subsequent
part (Step 5) in Section 5.3

5.2 3D Reconstruction of Initial Part

It is well known that the robustness of optimization-
based methods for 3D reconstruction from line draw-
ings is often sensitive to the initial settings of the \( z \)-
coordinates of the vertices [12]. There is no available
3D shape to guide 3D reconstruction from the initial
part \( G_{p_1} \). For robustness we independently reconstruct
a 3D shape from \( G_{p_1} \) with different initial settings:
randomized initialization of \( z \)-coordinates for multiple
times ([11], [12], [13]) and initial zero setting of \( z \)-
coordinates ([14]). The reconstruction result corresponding
to the minimum value of the objective function
\( \Phi(Z_{p_1}) = \sum_{j=1}^{N_c} w_j \phi_j(Z_{p_1}) \) (part of Equation (2)) is
selected as the final 3D shape of the initial part. In
our experiments, we find that ten randomizations plus
one zero setting are sufficient to obtain a good 3D
shape. It is worth mentioning that if the initial part is
a (quasi-)normalon [10] or has a cubic corner [10], [34],
[37], there exist more advanced algorithms to derive a
more effective initial approximation for the optimization
process.

5.3 3D Rough Shape Estimation of Subsequent
Parts

When the \( z \)-coordinates of the vertices of a line drawing
have been partially reconstructed, various image regular-

\[
IR_{lp} > IR_{lv} > IR_{c} > IR_{sda}
\]

As illustrated in Figs. 9(b)–(e), we use image regular-
\( IR_{lp} \), line verticality (\( IR_{lv} \)), and SDA
(standard deviation of the angles formed at a vertex equal
to zero, denoted as \( IR_{sda} \)) to estimate the depth of one
free vertex of \( G_{p_i} \). After the depth of one free vertex
is reconstructed, face planarity (Fig. 9(a)) can then be
used to derive the depths of the other vertices in this
part. It is possible that multiple image regularities are
in conflict with each other. To solve this problem, we
specify the priority of the four image regularities as
\( IR_{c} > IR_{lp} > IR_{lv} > IR_{sda} \). For instance, for a
free vertex whose depth can be derived by both \( IR_{lp} \)
and \( IR_{lv} \), we use line parallelism to estimate its depth.

In the implementation, we firstly select all the vertices
with unknown depths as free vertex candidates, which
connect to a part whose 3D shape has been reconstructed,
and then we check if the condition of any image regular-
ity is satisfied for one of the free vertex candidates in the
order of the priority. If one image regularity is satisfied,
we select the corresponding vertex as the free vertex
and compute its depth with this image regularity. Finally,
face planarity is exploited to derive the rough depths of
the other vertices in this part. Note that when \( IR_{lp} \)
is used to derive the depth of a free vertex, there usually
exist multiple reconstructed lines parallel to the line
containing the free vertex. In this case, the \( z \)-coordinate
of the free vertex is set to the average of the values
derived by these reconstructed lines.

6 Experiments

We conducted several experiments to evaluate the effi-
ciency and effectiveness of the proposed decomposition
and reconstruction algorithms. We focused on com-
parisons with the Divide-and-Conquer (DaC) method

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line drawings of increasing complexity, as shown successfully decomposed drawing). than that needed by DaC (about 5–14 seconds for each 0.01 second with our algorithm, and was much less decomposing each of these 30 line drawings was within is approximated by multiple planar faces. The time for handling curved-face manifolds if a curved surface decomposition results of the line drawings (27) beyond the predefined maximum search depth for DaC. The internal faces have more than 10 vertices, which is decomposed by our algorithm. In these line drawings, D could not be decomposed by DaC (with the maximum max set to 10), but were successfully algorithm was about four orders of magnitude faster than algorithm was about four orders of magnitude faster than D algorithm, DaC with Dmax = 10 and DaC with Dmax = 11, are illustrated by the three respective curves in Fig. 11(b). The results indicate that our algorithm is approximately linear in the number of line drawing parts, while DaC is largely exponential. In addition, on O7, our algorithm was about four orders of magnitude faster than DaC with Dmax = 11.

We conducted another experiment to evaluate the 3D reconstruction performance of the proposed method, including the decomposition step. Our method, PR for short, was compared to DaC and LS, the method by Lipson and Shpitalni [3]. LS is a typical 3D reconstruction method without line drawing decomposition. For fair comparison, the same reconstruction algorithm [3] was used for PR, DaC and LS.

The line drawings (1)–(24) in Fig. 10 were used in this experiment since the decomposition results by DaC and our algorithm were similar. The convergence criteria proposed by Liu et al. [13] since their method can handle more complex manifold objects than other previous methods. We implemented the proposed algorithms in C++, and ran them on a PC with an Intel(R) Dual Core(TM) i5 CPU M540@2.53GHz (only a single thread used for simplicity).

In the first experiment we tested our decomposition algorithm (Algorithm 1) and the decomposition algorithm in DaC (Steps 1 and 2 of Algorithm 3 in [13]) on 30 line drawings shown in the first and fourth columns of Fig. 10. Most of these line drawings were collected from previous papers such as [12], [13], and [38]. The DRFs of these line drawings vary from 4 to 17.

For the line drawings with indices (1)–(24), our algorithm obtained the decomposition results similar to those by DaC. Neither our algorithm (because their DRFs are already 4) nor DaC (because they have no internal faces) succeeded to decompose the two 4DRF line drawings (25) and (26). However, we will show in Section 7 that our algorithm can be extended to handle these two drawings by generating some new vertices and edges.

The line drawings (27–30) show four manifolds which could not be decomposed by DaC (with the maximum search depth Dmax set to 10), but were successfully decomposed by our algorithm. In these line drawings, the internal faces have more than 10 vertices, which is beyond the predefined maximum search depth for DaC. The decomposition results of the line drawings (27) and (28) indicate that our algorithm can be extended to handling curved-face manifolds if a curved surface is approximated by multiple planar faces. The time for decomposing each of these 30 line drawings was within 0.01 second with our algorithm, and was much less than that needed by DaC (about 5–14 seconds for each successfully decomposed drawing).

We further evaluated computational performance on line drawings of increasing complexity, as shown in Fig. 11. There are in total seven line drawings (Fig. 11(a)), indexed by seven loops O1−7 with Oi ⊂ Oi+1, 1 ≤ i ≤ 6. The times for decomposing O1−7 by our algorithm, DaC with Dmax = 10 and DaC with Dmax = 11, are illustrated by the three respective curves in Fig. 11(b). The results indicate that our algorithm is approximately linear in the number of line drawing parts, while DaC is largely exponential. In addition, on O7, our algorithm was about four orders of magnitude faster than DaC with Dmax = 11.

Fig. 9: Examples of estimating the unknown depths of vertices with image regularities. The vertices whose depths have been recovered are marked by □ in red and their 3D positions are used to estimate the depths of the vertices marked by □ in blue. (a) shows that the depths of the vertices v5, v6, v7, v8 can be computed with the 3D positions of v1, v2, v3, v4 using face planarity. In (b), the depth of the vertex v3 can be obtained with v1 and v2 using line collinearity. (c) shows how to estimate the depth of the vertex v10 using line parallelism, since edges {v1, v2}, {v3, v4}, {v5, v6}, and {v7, v8} are parallel with the edge {v9, v10} and the 3D positions of the vertices v1−9 have been recovered. (d) shows that the depth of the vertex v2 is estimated by line verticality, since the 3D shape O1 of the bottom part has been reconstructed. (e) shows a case in which the depth of the vertex v9 is estimated by SDA.

Fig. 11: (a) Seven line drawings of increasing complexity indexed by seven loops. (b) The times used to decompose the line drawings in (a) by our algorithm and DaC.

Fig. 10: Time (seconds)
Fig. 10: Thirty tested line drawings (1st and 4th columns), and their decomposition results by DaC (2nd and 5th columns) and our algorithm (3rd and 6th columns). Better viewed on the screen by enlarging the figure.
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for all the algorithms were the same: stopped when the difference between the values of the objective function in two consecutive iterations was below a given threshold (0.001 in our experiment). For each line drawing we ran each algorithm ten times. Two measurements were adopted to evaluate the robustness of the reconstruction process using these algorithms.

One is the average planarity error (APE) of all the faces of the ten reconstructed 3D object parts $O_{1-10}$ from a line drawing $\mathcal{L}$, defined as

$$APE(O_{1-10}|\mathcal{L}) = \frac{1}{10|\mathcal{L}|} \sum_{k=1}^{10} \frac{1}{N_f} \sum_{i=1}^{N_f} \sum_{j=1}^{M_{f_i}} D_{ij}^k,$$

where $D_{ij}^k = \frac{|a_j^k x_i^k + b_j^k y_i^k + c_j^k z_i^k + 1|}{\sqrt{(a_j^k)^2 + (b_j^k)^2 + (c_j^k)^2}}$ are the parameters of the best-fit plane for face $f_i$ obtained from the $k$th running (how to obtain the best-fit plane from a set of vertices can be found in [3]). $M_{f_i}$ is the number of vertices in face $f_i$, $\mathcal{L}$ denotes the size of the original line drawing, $\Delta L = \max\{|\Delta x, \Delta y, \Delta z|\}$ denotes the size of the faces in $\mathcal{L}$, $\Delta L$ is the width and $\Delta y$ is the height of $\mathcal{L}$, $N_f$ is the number of the faces in $\mathcal{L}$, and $(x_j^k, y_j^k, z_j^k)$ is the 3D coordinate of the $j$th vertex of face $f_i$ obtained from the $k$th running. The term $D_{ij}^k$ denotes the distance between vertex $(x_j^k, y_j^k, z_j^k)$ and the best-fit plane.

Another measurement is the average line-parallelism error (ALE) of all the parallel lines of the ten reconstructed 3D objects $O_{1-10}$, defined as

$$ALE(O_{1-10}|\mathcal{L}) = \frac{1}{10N_p} \sum_{k=1}^{10} \sum_{(i,j) \in S} \theta(\langle (l_i^k)^{3D}, (l_j^k)^{3D} \rangle),$$

where $S$ is the set of pairs of parallel lines $(l_i, l_j)$ in the line drawing which are recognized as parallel in 3D space by users, $N_p$ is the number of pairs in $S$, and $\theta(\langle (l_i^k)^{3D}, (l_j^k)^{3D} \rangle) = \cos^{-1}(\langle (l_i^k)^{3D}, (l_j^k)^{3D} \rangle)$ is the angle between two unit vectors $\langle (l_i^k)^{3D} \rangle$ and $\langle (l_j^k)^{3D} \rangle$ of two 3D parallel lines corresponding to $l_i$ and $l_j$, respectively, from the $k$th running. In this experiment, $S$ was obtained by these two steps: 1) find the set of all the pairs of parallel lines in the 2D line drawing (two lines were regarded as parallel if the angle between them was less than $\pi$); 2) then remove the pairs that were not interpreted as parallel by the users.

The quantitative evaluation results for these algorithms on the test examples are shown in Fig. 12. Fig. 12(a) indicates that PR is more efficient than the other two algorithms for all the examples. It is because PR computes initial 3D shapes which are already close to the optimal ones, thus greatly reducing the iterations of the optimization algorithm in reconstruction. It is shown in Figs. 12(b) and (c) that in most cases our method led to more robust reconstruction results than both DaC and LS. In fact large values of APE and ALE often signal distorted or failed reconstructions. Fig. 13 shows the reconstruction results from the first six line drawings of the test examples. It can be seen that for some examples, such as the one in the third column, although DaC could give roughly correct shapes from the decomposed parts, the integrated results might suffer from artifacts. This is mainly because DaC fails to faithfully maintain the globally geometrical relationships between parts during reconstruction.

Our algorithm is largely robust to sketching errors. Take the line drawing in Fig. 10(6) as an example. To analyze the sensitivity of our algorithm, we generated its random variations by randomly disturbing the 2D x- and y-coordinates of the vertices according to a Gaussian distribution $N(0, \sigma)$. Figs. 14(b–d) show three such noisy
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From our observations, we found that an algorithm can deal with a wide range of objects formed by 4DRF parts. Our decomposition algorithm can deal with a wide range of objects formed by 4DRF parts. From our observations, we found that a line drawing that can be well decomposed is required to satisfy two conditions. First, there are no missing edges and vertices in the line drawing. That is, the input line drawing must contain all the vertices and edges of the 4DRF manifolds forming the desired 3D object. Second, there must exist at least one trihedral vertex in the line drawing.

The second condition comes from the fact that in Step 3 of Algorithm 1, each selected pair (seed) of faces share a trihedral vertex. The line drawing shown in Fig. 15(a) is one without any trihedral vertex and thus cannot be decomposed by Algorithm 1. However, if a seed is not required to share a trihedral vertex (i.e., any two neighboring faces can be a seed), then Algorithm 1 can still decompose such line drawings. Fig. 14(b) gives such a result, which contains four decomposed parts \( p_{1-4} \). Obviously, a more desired decomposition is to have parts \( p_{1-3} \) combined into a single part. One possible solution to merge these non-manifold small parts into a 4DRF manifold is to use the information provided by \( p_4 \). From \( p_4 \), we can run the face identification algorithm in [28] on \( p_4 \) to obtain the new face \((1, 2, 3, 4, 5)\). Such face information makes the combination of \( p_{1-3} \) still a 4DRF manifold. In fact, we can always merge decomposed parts each with fewer faces into a larger line drawing, which may or may not represent a 4DRF manifold.

Therefore, to remove the second condition, the following four steps can be carried out: 1) run Algorithm 1 first; 2) check whether there is a part without any trihedral vertex; 3) if there is such a part, then on this part, run Algorithm 1 again but with a modified seed selection criterion, allowing any pair of neighboring faces as a seed; and 4) finally merge the parts whose numbers of faces are smaller than a threshold. Developing a complete merging method is our future work.

As shown in Figs. 10(25) and (26), a 4DRF object with trihedral vertices and many faces cannot be decomposed by Algorithm 1 or DaC. However, if suitable new edges and vertices are generated on the original line drawing, then the line drawing can still be decomposed into 4DRF parts. For example, in Fig. 15(c), if the dotted edges are added to the original line drawing represented by the solid lines, our algorithm could separate it into four cuboids (Fig. 15(d)). How to design a method to automatically add such edges and vertices is an interesting topic to explore further in the future.

Progressive 3D Reconstruction. The better performance of our approach comes from the estimate-and-optimize strategy for each subsequent part. There is still room for improvement in this strategy. The estimation of the initial 3D shape for one part may be improved by selecting the most suitable regularity depending on the nature of each part (such as using the axonometric inflation scheme in [10] for a quasi-normalon part). In
addition, a better strategy, which considers both the topological relationship between parts and the reconstruction quality of each part, can be utilized to determine a better reconstruction sequence of all the decomposed parts.

3D Beautification. After a 3D object is reconstructed by our algorithm (or other algorithms), it is possible to use a beautification algorithm to improve the result. There exist a few methods [39], [40], [41] for this task, in which the goal is to refine the x-, y-, and z-coordinates of all the 3D vertices for example through constrained optimization. It should be mentioned that if a reconstructed object is too distorted, a 3D beautification algorithm cannot help much usually; developing a good reconstruction algorithm is more crucial.

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